

1st GIMO 2021

Gaussian Curvature

Day I Problems

Problem 1. In an acute-angled triangle ABC with $AB \neq AC$, let P, Q be points on the segment BC , such that $CA = CP$ and $BA = BQ$. Let ω be the circumcircle of triangle ABC and AP, AQ intersect ω at points X and Y , other than A . The perpendiculars from X, Y onto line BC meet ω at X', Y' , respectively.

How many pairs of points (M, N) (with M on AP and N on AQ) exist such that line MN is parallel to BC , and points M, N, X', Y' lie on the same circle?

Problem 2. A crazy biologist has $n \geq 3$ Petri dishes arranged in a line. Initially, some of them contain one bacterium each, while the rest are empty. The biologist then performs the following experiments:

Firstly, he makes each bacterium divide into 2, and in each of the resulting pairs, one bacterium remains in its Petri dish and the other moves to the next one.

Then, if some Petri dish contains at least 2 bacteria, the conditions there become unsuitable for living, and thus, one of them dies while one of them moves to the next Petri dish. In any case, a bacterium that leaves the last Petri dish dies.

The experiment ends when the conditions in all Petri dishes are suitable for living.

Help the biologist find the least positive integer k (which may depend on n) such that after k experiments, the arrangement of bacteria in the Petri dishes is always the same as before experiments.

Problem 3. Determine all functions f mapping positive reals to positive reals such that

$$f(x)f(x + 2f(y)) = xf(x + y) + f(x)f(y),$$

for all positive reals x, y .

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Submission Process I. You can submit your solutions on AoPS PM to [Aritra12](#) and [Orestis_Lignos](#) if you have an account at AoPS. This is the most preferred way of submission and it is also beneficial to participants because on AoPS PM you are allowed to send solutions one by one in that single PM however you are not allowed so for the other two processes. But obviously you can send day 1 and day 2 separately.

Submission Process II. For submitting Day I Solutions please upload solutions in the following link: <https://forms.gle/EpcqUQFhZhzbch8PA>

Submission Process III. If you are unable to do any of the things above then just simply mail your solutions pdf to us on our mail gaussiancurv180@gmail.com